International Journal of Theoretical Physics, Vol. 15, No. 10 (1976), pp. 773–779

New Theoretical Approach Concerning the Physical Phenomena in Space and Time, and the Action Principle

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Received: 26 September 1975

Abstract

The unity of physical nature is advanced. A primordial continuum with geometric character is assumed to exist; its coordinates are the elements generating the physical magnitudes. The relation between these primordial coordinates and the physical magnitudes is established. Starting from the condition of minimum path in the primordial continuum, an explanation of the action principle is given. In sequence the equations for the four-dimensional space-time are obtained, and with some specializations the equation of gravity is derived.

1. *Basic Concepts*

By defining an absolute space with Euclidean character, Newton (1726) expressed in a clear mathematical form some fundamental laws of nature. Later on it was shown that the space constituted by the coordinates entering the equations of physics is not absolute, and its character deviates from the Euclidean geometry (Einstein, 1955). But, as space is a primary notion (Fock, 1960), this means that, by introducing certain "universal" constants from the beginning, some basic "preferences" would exist in the very essence of nature. We think this is unacceptable. Our idea is to enlarge the notion of physical space.

To start with we admit a fundamental unity of physical nature, by the following proposition: All magnitudes describing the physical phenomena are generated from a unique objective existence. According to the ideas of Becker (1959) and Fock (1960), as well as of those physicists who want a geometrization of all fields, we assume that the above primary existence is a continuum with geometric character. It has a Euclidean geometry, and it must include both space and time, as well as the magnitudes characterizing the physical fields. We could call it "proto-space-time-field"; in order to avoid a new name we shall refer to this geometric primary existence as GPE. Still the GPE coordinates cannot be identified simply with the space coordinates and the other physical magnitudes, but they are linked by a dynamical relation as follows. Let $\alpha_\epsilon(\epsilon = 1, 2, 3, \ldots)$ be the GPE coordinates. On the other hand let z_ι be some variables associated to a point in the space of the phenomena (SP), ζ_κ some variables associated to a moment of time, and $\overline{\overline{W}}_\lambda$ absolute magnitudes characterizing the physical phenomena, not necessarily observables, but serving to the construction of the observables. To each "point" in the GPE there are corresponding certain functionals Ω_φ depending in general on z_ι, ζ_κ, $\overline{\overline{W}}_\lambda$, and their derivatives. In order to make a mathematical correspondence, a cross section Ξ in the GPE is needed, with a number of dimensions equal to the dimensions of the SP, plus the time, plus the number of the functionals Ω_φ. If $\beta_\rho{}^\Xi$ are intrinsic GPE coordinates, then to a neighboring "point" $\beta_\rho{}^\Xi + \Delta\beta_\rho{}^\Xi$ on the GPE cross section there are corresponding the functionals $\Omega_\varphi + \Delta\Omega_\varphi$ in a neighboring point of the SP, and in a neighboring moment of time. Thus the physical phenomenon, that is, the passage from one physical state to another, corresponds to a passage from one GPE coordinate to another. Therefore a relation between the increments of the GPE coordinates and the increments in the SP must be expressed: The most natural and the simplest expression is

$$\Delta\beta_\rho{}^\Xi = \epsilon_\rho{}^\Xi \prod_\varphi \Delta\Omega_\varphi{}^\Xi \prod_\iota \Delta z_\iota{}^\Xi \prod_\kappa \Delta\zeta_\kappa{}^\Xi \tag{1.1}$$

The dynamic character of the relation is put forth by the entities e_ρ. These entities indicate the engendering by the GPE of the quantities describing the physical phenomena. From a mathematical point of view the e_ρ – s are operators, operating specifically on all quantities on the right-hand side. Postulating different kind of operations one can obtain various physical theories, which are approximations of different extent.

Now, the finite physical phenomenon is described by a "line" between two "points" in GPE. It is a natural presumption that the actual physical phenomenon corresponds to the shortest "path" between the two "points", i.e.,

$$\int_1^2 d\sigma = \text{minimum} \tag{1.2}$$

where $d\sigma$ is the element of "length" in the GPE, given by

$$d\sigma^2 = \sum_\rho \sum_\tau \gamma_{\rho\tau}^\Xi d\beta_\rho{}^\Xi d\beta_\tau{}^\Xi \tag{1.3}$$

$\gamma_{\rho\tau}^\Xi$ being the transformation coefficients of α_ϵ into $\beta_\rho{}^\Xi$.

The differentials $d\beta_\rho{}^\Xi$ are obtained by the passage to the limit of the expression (1.1). This can be performed as follows. Since the variables z_ι and ζ_κ determine a point in space at a moment of time, we can say they are independent of the GPE cross section, and Δz_ι^Ξ and $\Delta\zeta_\kappa{}^\Xi$ become the differentials dx^a and $d\theta$, x^a being the space coordinates and θ a time parameter (Dorling, 1970). As for $\Delta\Omega_\varphi{}^\Xi$, we can assert that they are increments with respect to the variables x^a and θ by means of the magnitudes $\bar{\bar{W}}_\lambda$; to the limit we denote them with $\tilde{d}\Omega_\varphi{}^\Xi$. Thus equation (1.1) becomes

$$d\beta_\rho{}^\Xi = e_\rho{}^\Xi \prod_\varphi \tilde{d}\Omega_\varphi{}^\Xi \prod_a dx^a d\theta \tag{1.4}$$

To satisfy the condition (1.2) we must have

$$\delta \int_1^2 \prod_a dx^a d\theta \left[\sum_\rho \sum_\tau \gamma_{\rho\tau}^\Xi \left(e_\rho{}^\Xi \prod_\varphi \tilde{d}\Omega_\psi{}^\Xi \right) \left(e_\tau{}^\Xi \prod_\psi \tilde{d}\Omega_\psi{}^\Xi \right) \right]^{1/2} = 0 \tag{1.5}$$

the integral being evidently multiple, over x^a, θ, and perhaps other variables. Now, if the square root is considered as a Lagrangian density, one sees that equation (1.5) constitutes the action principle. We must just identify some expressions in the integrand with well-known physical magnitudes. So far, besides formal reasons, there was not a natural explanation for the utilization of the action principle as common starting point of several theories. Now the general action principle can be interpreted as principle of minimum path in the GPE, that is, in the primordial form of the space. The "action" of different phenomena is interpreted as a "length" in the GPE, expressed by means of observable physical quantities.

2. *Four-Dimensional Space-Time*

We shall use the principle (1.5) to obtain the equations for the four-dimensional space-time. This will be performed by some natural considerations and postulating some particular operations of the e_ρ's.

One knows that being given a Lagrange function (Lagrange density) $\mathscr{L}$, depending on generalized coordinates and their derivatives up to higher order, the action principle allows one to obtain the Euler–Lagrange equations with higher derivatives (Borneas, 1960, 1969). In this case the generalized coordinates are associations of the magnitudes $\bar{\bar{W}}_\lambda$ and other magnitudes appearing in $\tilde{d}\Omega_\varphi{}^\Xi$, with the e_ρ's. As the square root in (1.5) comprises various physical phenomena, in order to put it into the form of a sum we write

$$\left(\sum_\rho \chi_\rho \prod_\varphi \tilde{d}\Omega_\varphi \right)^2 \equiv \sum_\rho \sum_\tau \gamma_{\rho\tau} \left(e_\rho \prod_\varphi \tilde{d}\Omega_\varphi \right) \left(e_\tau \prod_\psi \tilde{d}\Omega_\psi \right) \tag{2.1}$$

(the index Ξ has been dropped for simplicity), with the condition

$$\chi_\rho \chi_\tau + \chi_\tau \chi_\rho = 2\gamma_{\rho\tau} e_\rho e_\tau \tag{2.2}$$

Thus we have

$$\mathscr{L} = \sum_\rho \left(\chi_\rho \prod_\varphi \tilde{d}\Omega_\varphi \right) \tag{2.3}$$

the χ_ρ's containing the e_ρ's.

If the combination of the three-dimensional space (Freeman, 1969) and of the time into a unique continuum is accepted, we include x^a and θ into the notation x^j ($j = 0, 1, 2, 3$), and we can write $\Pi_a dx^a\, d\theta = \Pi_j dx^j = dx$. Of course more dimensions are also possible (Klein, 1926; Souriau, 1963; Rayski, 1965; Borneas, 1972).

The metric of the four-dimensional space-time is given by

$$ds^2 = \sum_h \sum_j g_{hj} dx^h\, dx^j \tag{2.4}$$

Equation (1.5) becomes

$$\delta \int_1^2 dx \sum_\rho \left(\prod_\varphi \tilde{d}\Omega_\varphi \right)^{\hat{\rho}} \tag{2.5}$$

(the index $\hat{\rho}$ shows that the χ_ρ's operate on the magnitudes inside the parenthesis) and the increments $\tilde{d}\Omega_\varphi$ have the form (from now on the summation rule will be used)

$$\tilde{d}\Omega_\varphi = [\partial\Omega/\partial(W_\lambda)_{\eta(K)u\cdots}]D(W_\lambda)_{\eta(K)u\cdots} \tag{2.6}$$

where $(W_\lambda)_\eta$ are the components of the magnitudes $\overline{\overline{W}}_\lambda$; the index $(K)u\cdots$ shows the Kth-order derivative with respect to $x^u, x^v, \cdots$; D means the absolute (covariant) differential.

Consider now the three-dimensional space embedded in the N-dimensional GPE cross section; Ghosh (1973, 1974) has considered a fourfold embedded in a N space, but in our present approach we prefer to emphasize the special nature of the time. The simplest case for the GPE cross section is to be Euclidean with $N = 3\,(3 + 1)/2 = 6$ dimensions. This leads to two functionals Ω_φ.

We obtain the equations concerning the space-time if in equation (2.5) only the variations with respect to g^{hj} is maintained. The quantities g^{hj} being tensors of second order, we admit that $\overline{\overline{W}}_\lambda$ are also tensors of second order at the most. Then equation (2.5) can be written

$$\begin{aligned}\delta_g \int_1^2 dx \sum_\rho \big(\{(\partial\Omega_1/\partial W^{kl}_{\lambda(K)u\cdots})[(\partial W^{kl}_{\lambda(K)u\cdots}/\partial x^p) + \Gamma^k_{qp} W^{ql}_{\lambda(K)u\cdots} \\ + \Gamma^l_{qp} W^{kq}_{\lambda(K)u\cdots}]\,dx^p\}\{(\partial\Omega_2/\partial W^{mn}_{\mu(L)v\cdots})[(\partial W^{mn}_{\mu(L)v\cdots}/\partial x^r) \\ + \Gamma^m_{sr} W^{sn}_{\mu(L)v\cdots} + \Gamma^n_{sr} W^{ms}_{\mu(L)v} + \ldots]\,dx^r\}\big)^{\hat{\rho}} = 0\end{aligned} \tag{2.7}$$

Γ^k_{qp} being the Christoffel's symbols. Multiplying and dividing in equation (2.7) by $(d\omega)^2$, ω being a certain parameter, and multiplying the parentheses we obtain

$$\begin{aligned}\delta_g \int_1^2 dx \left\{ \sum_\rho \left[\iint_1^2 (d\omega)^2 (dx^p/d\omega)(dx^r/d\omega)(\partial\Omega_1/\partial W^{kl}_{\lambda(K)u\cdots}) \right.\right. \\ \left. \times (\partial\Omega_2/\partial W^{mn}_{\mu(L)v\cdots})(\partial W^{kl}_{\lambda(K)u\cdots}/\partial x^p)(\partial W^{mn}_{\mu(L)v\cdots}/\partial x^r) \right|^{\hat{\rho}}\end{aligned}$$

$$+\sum_{\tau}\left[\iint_1^2 (d\omega)^2(dx^p/d\omega)(dx^r/d\omega)(\partial\Omega_1/\partial W^{kl}_{\lambda(K)u}\dots)\right.$$

$$\left.\times(\partial\Omega_2/\partial W^{mn}_{\mu(L)v}\dots)(\partial W^{kl}_{\lambda(K)u}\dots/\partial x^p)W^{sn}_{\mu(L)v}\dots\Gamma^m_{sr}\right]^{\hat{\tau}}$$

$$+\sum_{\tau}\left[\iint_1^2 (\text{same as above})(\partial W^{kl}_{\lambda(K)u}\dots/\partial x^p)W^{ms}_{\mu(L)v}\dots\Gamma^n_{sr}\right]^{\hat{\tau}}$$

$$+\sum_{\tau}\left[\iint_1^2 (\text{same as above})(\partial W^{mn}_{\mu(L)v}\dots/\partial x^r)W^{sl}_{\lambda(K)u}\dots\Gamma^k_{sp}\right]^{\hat{\tau}}$$

$$+\sum_{\tau}\left[\iint_1^2 (\text{same as above})(\partial W^{mn}_{\mu(L)v}\dots/\partial x^r)W^{ks}_{\lambda(K)u}\dots\Gamma^l_{sp}\right]^{\hat{\tau}}+\dots$$

$$+\sum_{\pi}\left[\iint_1^2 (d\omega)^2(dx^p/d\omega)(dx^r/d\omega)(\partial\Omega_1/\partial W^{kl}_{\lambda(K)u}\dots)\right.$$

$$\left.\times(\partial\Omega_2/\partial W^{mn}_{\mu(L)v}\dots)W^{ql}_{\lambda(K)u}\dots W^{sn}_{\mu(L)v}\dots\Gamma^k_{qp}\Gamma^m_{sr}\right]^{\hat{\pi}}$$

$$+\sum_{\pi}\left[\iint_1^2 (\text{same as above})W^{ql}_{\lambda(K)u}\dots W^{ms}_{\mu(L)v}\dots\Gamma^k_{qp}\Gamma^n_{sr}\right]^{\hat{\pi}}$$

$$+\sum_{\pi}\left[\iint_1^2 (\text{same as above})W^{kg}_{\lambda(K)u}\dots W^{sn}_{\mu(L)v}\dots\Gamma^l_{qp}\Gamma^m_{sr}\right]^{\hat{\pi}}$$

$$\left.+\sum_{\pi}\left[\iint_1^2 (\text{same as above})W^{kg}_{\lambda(K)u}\dots W^{ms}_{\mu(L)v}\dots\Gamma^l_{qp}\Gamma^n_{sr}\right]^{\hat{\pi}}+\dots\right\}$$

$$=\delta_g\int_1^2 dx\,\{\mathscr{L}_E(\rho)+\mathscr{L}_F(\tau)+\mathscr{L}_G(\pi)\}=0 \tag{2.8}$$

the expressions $\mathscr{L}_E(\rho)$, $\mathscr{L}_F(\tau)$, $\mathscr{L}_G(\pi)$ being obvious.

In order to write down laws concerning the space-time, we must make visible the Riemann curvature tensor (keeping first derivatives only)

$$R^k_{spr}=(\partial\Gamma^k_{sr}/\partial x^p)-(\partial\Gamma^k_{sp}/\partial x^r)+(\Gamma^k_{qp}\Gamma^q_{sr}-\Gamma^k_{qr}\Gamma^q_{sp}) \tag{2.9}$$

To achieve this we put (keeping first derivatives only)

$$\sum_{\pi}\left[\iint_1^2 (d\omega)^2(dx^p/d\omega)(dx^r/d\omega)(\partial\Omega_1/\partial W^{kl}_{\lambda})(\partial\Omega_2/\partial W^{mn}_{\mu})\right.$$

$$\left.\times W^{ql}_{\lambda}W^{sn}_{\mu}\right]^{\hat{\pi}}=G^{\mathrm{I}}(prqs\bar{i}) \tag{2.10}$$

where $\tilde{i}$ means all indices resulting from the derivatives of the Ω_1 and Ω_2, and existing also in the Christoffel's symbol. Thus

$$\mathscr{L}_G = G^{\mathrm{I}}(prqskm)\Gamma^k_{qp}\Gamma^m_{sr} + G^{\mathrm{II}}(prqskn)\Gamma^k_{qp}\Gamma^n_{sr} + G^{\mathrm{III}}(prqslm)\Gamma^l_{qp}\Gamma^m_{sr} + G^{\mathrm{IV}}(prqsln)\Gamma^l_{qp}\Gamma^n_{sr} \tag{2.11}$$

The magnitudes G can be considered as representing a field contributing to the determination of the space-time structure. We postulate certain symmetries of the field G. Changing the notation of some indices, admitting the relation

$$-G^{\mathrm{I}}(prqskm) - G^{\mathrm{II}}(prqskm) = G^{\mathrm{III}}(rpqskm) + G^{\mathrm{IV}}(rpqskm) = G(prqskm)\delta_{qm} \tag{2.12}$$

and furthermore the independence on the index q, we obtain

$$\mathscr{L}_G = G(prsk)[\Gamma^k_{qp}\Gamma^q_{sr} - \Gamma^k_{qr}\Gamma^q_{sp}] \tag{2.13}$$

according to equation (2.9) $\mathscr{L}_G$ contains the Riemann tensor and the derivatives of the Christoffel's symbols.

In a similar way $\mathscr{L}_F$ can be expressed by the Christoffel's symbols and a field F, leading straightforwardly to an expression of the metric tensor. In fact, with

$$\sum_{\tau}\left[\int\int_1^2 (d\omega)^2(dx^p/d\omega)(dx^r/d\omega)(\partial\Omega_1/\partial W^{kl}_{\lambda(K)u}\ldots)(\partial\Omega_2/\partial W^{mn}_{\mu(L)v}\ldots) \times (\partial W^{kl}_{\lambda(K)u}\ldots/\partial x^p)W^{sn}_{\mu(L)v}\ldots\right]^{\hat{\tau}} = F^{\mathrm{I}}(rsm) \tag{2.14}$$

changing some indices, putting $F^{\mathrm{I}} + F^{\mathrm{II}} + F^{\mathrm{III}} + F^{\mathrm{IV}} = F(rsm)$, and then making $m = r$ one obtains (Landau and Lifshitz, 1960)

$$\mathscr{L}_F = F(rs)\Gamma^r_{sr} = F(rs)g^{rn}\partial g_{rn}/\partial x^s \tag{2.15}$$

Equation (2.8) now reads

$$\delta_g \int_1^2 dx\{\mathscr{L}_E + F(rs)g^{rn}\partial g_{rn}/\partial x^s + G(prsk)[\Gamma^k_{qp}\Gamma^q_{sr} - \Gamma^k_{qr}\Gamma^q_{sp}]\} = 0 \tag{2.16}$$

Consequently it is seen that the contraction with respect to indices k and r of the covariant curvature tensor leads to the Einstein tensor R_{sp}; therefore we take into consideration another specialization of the field G:

$$G(prsk) = G(prsk)\delta_{kr} \tag{2.17}$$

(no summation over k, r). On the other hand one knows (Landau and Lifshitz, 1960) that

$$\delta_g \int_1^2 g^{1/2}\, dx g^{ps}(\Gamma^k_{qp}\Gamma^q_{sk} - \Gamma^q_{sp}\Gamma^k_{qk}) = \delta_g \int_1^2 g^{1/2}\, dx R \tag{2.18}$$

R being the scalar curvature of the space-time. Consider G independent on the index k, and make the identification

$$G(ps) = g^{ps} y g^{1/2} \tag{2.19}$$

then from equation (2.16) it results

$$R_{hj} - g_{hj}R/2 + y^{-1}g^{-1/2}\delta\,[F(rs)g^{rn}\partial g_{rn}/\partial x^s]\,/\delta g^{hj} + y^{-1}g^{-1/2}\delta\,\mathscr{L}_E/\delta g^{hj} = 0 \tag{2.20}$$

If the last term from above is related to the energy-momentum tensor of the matter, then equation (2.20) is the equation of the space-time-gravity. It differs from the Einstein equation by the third term in equation (2.20), which corresponds to a supplementary field; this term is most likely small in usual cases, but we think it might be observable in case of very rapid variations of the gravitational field. It is up to experimental research to seek this second gravitational field.

3. *Conclusions*

The present paper gives a unified explanation to some basic features of the space and fields, and to the action principle. But there is room for a program to derive other physical laws from equation (1.5) or (2.5), as a result of how the e_ρ's operate, and taking into consideration other variations too. It may be that some associations of the magnitudes $\overline{\overline{W}}_\lambda$ with the e_ρ's could lead to quantities with quantal character. There are a lot of possibilities, but, of course, more principles must be introduced. Nevertheless the important fact is the existence of the principle of minimum "path" in the GPE as a common root.

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